Triplectic Quantization of W2 gravity

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Abstract

The role of one loop order corrections in the triplectic quantization is discussed in the case of W2 theory. This model illustrates the presence of anomalies and Wess Zumino terms in this quantization scheme where extended BRST invariance is represented in a completely anticanonical form.

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1   Introduction

The so called triplectic quantization[1, 2, 3] is a general Lagrangian gauge theory quantization procedure following the general lines of the field antifield or Batalin Vilkovisky (BV) method[4, 5] but with the requirement of extended BRST[6, 7, 8] (BRST plus anti-BRST) invariance rather than just BRST. In the usual BV quantization the BRST invariance is translated into the so called master equation. At zero loop order this equation is well defined and its solution, together with the appropriate requirements corresponding to gauge fixing, leads to the construction of the complete structure of ghosts, antighosts, ghosts for ghosts, etc[9, 10]. At higher orders in $\hbar$ one needs however to introduce some regularization procedure in order to give a well defined meaning to the mathematical objects involved in the formal master equation. Anomalies and Wess Zumino terms can this way be calculated at one loop order[11, 12].

In the triplectic quantization the extended BRST invariance is translated into a set of two master equations corresponding to the requirements of BRST and anti-BRST invariances respectively. As in the standard BV case, both equations have formally an expansion in loop order. One then expects that anomalies and Wess Zumino terms should show up at one loop order as long as one is able to introduce appropriate regularization schemes. These features are not present in the recently discussed case of Yang Mills theory[13]. In that case only the zero loop order corrections are relevant, as there are no anomalies. The important features of calculation of anomalies and counterterms in the triplectic context have not yet been discussed in the literature. In this article we will discuss the W2 model where the one loop order corrections will nicely illustrate the behavior of the quantum master equations, compared with the standard BV case. We will also show how to fix the gauge by means of canonical transformations.

2   Triplectic quantization

Considering some gauge theory, we enlarge the original field content ϕ^i, adding all the usual gauge fixing structure: ghosts, antighosts and auxiliary fields associated with the original gauge symmetries. The resulting set will be denoted as ϕ^A. Then we associate with each of these fields five new quantities, introducing the sets: $\bar{\phi}^A$, ϕ_A^{*1}, ϕ_A^{*2}, π_A^1 and π_A^2. The Grassmanian parities of these fields are: $\epsilon(\phi^A) = \epsilon(\bar{\phi}^A) \equiv \epsilon_A$, $\epsilon(\phi_A^{*a}) = \epsilon(\pi_A^a) = \epsilon_A + 1$. In this way the ideas of extended BRST quantization in the antifield context previously discussed in[14, 15, 16] are put in a completely anticanonical setting. The extended BRST invariance of the generating functional, defined on this 6n dimensional space, is equivalent to the fact that the quantum action W is a solution of the two master equations:

$$\frac{1}{2}\{W, W\}^a + V^a W = i\hbar \Delta^a W \tag{1}$$

where the indices $a = 1, 2$ correspond respectively to BRST and anti-BRST invariances and the extended form of the antibrackets, triangle and V operators read

$$\{F, G\}^a \equiv \frac{\partial^r F}{\partial \phi^A} \frac{\partial^l G}{\partial \phi_A^{*a}} + \frac{\partial^r F}{\partial \bar{\phi}^A} \frac{\partial^l G}{\partial \pi_A^a} - \frac{\partial^r F}{\partial \phi_A^{*a}} \frac{\partial^l G}{\partial \phi^A} - \frac{\partial^r F}{\partial \pi_A^a} \frac{\partial^l G}{\partial \bar{\phi}^A} \tag{2}$$

$$\Delta^a \equiv (-1)^{\epsilon_A} \frac{\partial^l}{\partial \phi^A} \frac{\partial^l}{\partial \phi_A^{*a}} + (-1)^{\epsilon_A} \frac{\partial^l}{\partial \bar{\phi}^A} \frac{\partial^l}{\partial \pi_A^a} \tag{3}$$

$$V^a = \frac{1}{2}\epsilon^{ab} \left(\phi^*_{Ab} \frac{\partial^r}{\partial \bar{\phi}^A} - (-1)^{\epsilon_A} \pi_{Ab} \frac{\partial^r}{\partial \phi^A} \right) . \tag{4}$$

here and in the rest of the article, unless explicitly indicated, we are adopting the convention of summing over repeated indices.

The Vacuum functional is normally defined including also an extra functional X

$$Z = \int [\mathcal{D}\phi][\mathcal{D}\phi^*][\mathcal{D}\pi][\mathcal{D}\bar{\phi}][\mathcal{D}\lambda] \, exp\{\frac{i}{\hbar}(W + X)\} \tag{5}$$

that represents gauge fixing and must satisfy the equations

$$\frac{1}{2}\{X \, , \, X\}^a - V^a X = i\hbar\Delta^a X \tag{6}$$

An alternative way of gauge fixing, using canonical transformations rather than including the functional X was proposed in [13]. We will use this method in section **4** for gauge fixing W2 theory.

Expanding the quantum action in powers of $\hbar$: $W = S + \hbar M_1 + ...$ we can look at the two first orders of the master equations

$$\frac{1}{2}\{S \, , \, S\}^a + V^a S = 0$$
$$\{S \, , \, M_1\}^a + V^a M_1 = i\Delta^a S \tag{7}$$

For a gauge theory with closed and irreducible algebra, corresponding to a classical action $S_0[\phi^i]$, a solution for the zero loop order action S is:

$$S = S_0 + \phi^{*\,a}_A \, \delta_a\phi^A + \frac{1}{2}\bar{\phi}_A\delta_2\delta_1\phi^A + \frac{1}{2}\epsilon^{ab}\phi^*_{A\,a} \, \pi^A_b \tag{8}$$

where the δ_a represent gauge fixed BRST ($a = 1$) and anti-BRST ($a = 2$) transformations of the fields (in other words, for theories with closed algebra, the standard BRST extended algebra associated with the gauge theory). In this article we will not be dealing with the generalized BRST transformations of the triplectic formalism[1, 2, 3] but just with standard transformations that do not involve the antifields.

Let us consider now the one loop order equation. As it happens in the standard BV case, we need to introduce a regularization procedure in order to give a well defined meaning to the operator $\Delta^a S$ as there are two functional derivatives acting on the same space time point. If we consider actions of the form (8) we see that the second term in the Δ^a operator will not contribute and the important term in the action is just $\phi^{*\,a}_A \, \delta_a\phi^A$. That means we must regularize:

$$\frac{\partial^l}{\partial\phi^A} \frac{\partial^l}{\partial\phi^{*\,a}_A} \left(\phi^{*\,a}_A \, \delta_a\phi^A \right) . \tag{9}$$

If the BRST algebra is such that the BRST and anti-BRST transformations are symmetrical, just changing ghosts by antighosts, then the same regularization can be used in both sectors. Moreover, we can use the same regularization used in the standard BV quantization.

3 Extended BRST invariance in W2 gravity

The classical action corresponding to W2 gravity reads [12]

$$S_0 = \frac{1}{2\pi} \int d^2x \left(\partial\phi\bar{\partial}\phi - h(\partial\phi)^2 \right) \tag{10}$$

and the corresponding BRST anti BRST algebra, satisfying $(\delta_1)^2 = (\delta_2)^2 = \delta_1\delta_2 + \delta_2\delta_1 = 0$ is

$$
\begin{aligned}
\delta_1\phi &= c_1\partial\phi \\
\delta_1 h &= \bar{\partial}c_1 - h\partial c_1 + \partial h c_1 \\
\delta_1 c_1 &= \partial c_1\, c_1 \\
\delta_1 c_2 &= b \\
\delta_2\phi &= c_2\partial\phi \\
\delta_2 h &= \bar{\partial}c_2 - h\partial c_2 + \partial h c_2 \\
\delta_2 c_1 &= -b - c_2\partial c_1 - c_1\partial c_2 \\
\delta_2 c_2 &= \partial c_2\, c_2
\end{aligned}
\tag{11}
$$

where we are representing BRST and anti-BRST transformations respectively as δ_1 and δ_2.

The general form of the gauge fixed action, after functionally integrating over the auxiliary fields of the triplectic formalism: $\bar{\phi}^A$, ϕ_A^{*a} and π_A^a is

$$S = S_0 + \delta_1\delta_2 B \tag{12}$$

where $B[\phi^A]$ is a bosonic functional. Therefore the ultimate result of triplectic quantization would be to build up such an object. However it is not possible to find a bosonic functional B that removes the degeneracy of the action S_0 using just the fields of algebra (11). We need more fields in order to obtain such a gauge fixing in W2 theory. Inspired in the extended algebra for the bosonic string from ref. [17] we can introduce the bosonic fields L and λ and the fermionic fields η and $\bar{\eta}$ and try transformations of the form

$$
\begin{aligned}
\delta_1\eta &= \partial\eta c_1 + 2\alpha\partial c_1\eta \\
\delta_1 L &= a_1\eta + \partial L c_1 - 2\alpha\partial c_1 L \\
\delta_1\lambda &= \partial\lambda c_1 - 2\alpha\partial c_1\lambda \\
\delta_1\bar{\eta} &= a_2\lambda + \partial\bar{\eta}c_1 + 2\partial c_1\bar{\eta} \\
\delta_2\eta &= b_1\lambda + \partial\eta c_2 + 2\alpha\partial c_2\eta \\
\delta_2 L &= b_2\bar{\eta} + \partial L c_2 - 2\alpha\partial c_2 L \\
\delta_2\lambda &= \partial\lambda c_2 - 2\alpha\partial c_2\lambda \\
\delta_2\bar{\eta} &= \partial\bar{\eta}c_2 + 2\alpha\partial c_2\bar{\eta}
\end{aligned}
\tag{13}
$$

$$\tag{14}$$

Extended nilpotency is satisfied if $a_1 b_1 + a_2 b_2 = 0$ for any α.

4

We will choose $a_1 = a_2 = b_1 = -b_2 = 1$ and $\alpha = 1$.

In this enlarged space we can choose the gauge fixing boson as

$$B = L(h - \tilde{h}) \tag{15}$$

where $\tilde{h}$ is a BRST anti-BRST invariant background field.

If we redefine the fields as

$$
\begin{aligned}
\lambda' &= -\lambda + \partial\bar{\eta}c_1 + 2\partial c_1\bar{\eta} + \partial Lb - \partial\eta c_2 - \partial^2 Lc_1 c_2 - \partial L\partial c_1 c_2 \\
&+ \partial^2 c_1 Lc_2 + 2\partial c_1 \partial Lc_2 - 2\partial c_2\eta - 2\partial c_2\partial Lc_1 + 4\partial c_2\partial c_1 L - 2\partial bL \\
\eta' &= \eta + \partial Lc_1 - 2\partial c_1 L \\
b' &= \bar{\eta} + L\partial c_2
\end{aligned} \tag{16}
$$

the gauge fixing action gets

$$\delta_1\delta_2\Big(L(h - \tilde{h})\Big) = \lambda'(h - \tilde{h}) + b'(\bar{\partial}c_1 - h\partial c_1 + \partial hc_1) + \eta'(\bar{\partial} - h\partial + \partial h)c_2 + L(\bar{\partial} - h\partial + \partial h)b \ . \tag{17}$$

The two last terms cancel each other by a supersymmetric compensation in the path integral[17] while the remaining two first terms correspond to the gauge fixing obtained in ref.[12] in the standard BV scheme with just BRST invariance. Thus, the boson B of eq. (15) would appropriately fix the gauge of W2 gravity. Now we will see in the next section how to arrive at this gauge fixing action of eq. (17) starting from the triplectic action.

4   Gauge fixing by canonical transformations

One interesting way to get the gauge fixed action of the form $S = S_0 + \delta_1\,\delta_2\,B$ from the triplectic action (8) is to perform canonical transformations in the triplectic space. These transformations have been studied in [13]. For each of the antibrackets of eq. (2) with $a = 1, 2$ we introduce a generator $F_a\,[\phi^A, \bar{\phi}^A, \phi_A^{*a\prime}, \pi_A^{a\prime}\,]$ and write out the set of transformations

$$
\begin{aligned}
\phi^{A\prime} &= \frac{\partial F_a}{\partial\phi_A^{*a\prime}} \\
\phi_A^{*a} &= \frac{\partial F_a}{\partial\phi^A} \\
\bar{\phi}^{A\prime} &= \frac{\partial F_a}{\partial\pi_A^{a\prime}} \\
\pi_A^a &= \frac{\partial F_a}{\partial\bar{\phi}^A} \ ,
\end{aligned} \tag{18}
$$

where there is no sum over a. If the matrix

$$T_a^{\alpha\,\beta} = \frac{\partial^r\partial^r F_a}{\partial z_\alpha^{*a\prime}\partial z_\beta} \tag{19}$$

(where there is again no sum over a and we are defining $\{z^\alpha\} \equiv \{\phi^A, \bar{\phi}^A\}$ and $\{z_\alpha^{*a\prime}\} \equiv \{\phi_A^{*a\prime}, \pi_A^{a\prime}\}$) is invertible, each of these transformations, for fixed $a = 1$ or 2 will not change the form of the corresponding antibracket.

If additionally the generators F_1 , F_2 both with non singular matrices (19) satisfy also the constraints

$$\frac{\partial F_1}{\partial \phi_A^{*1\prime}} = \frac{\partial F_2}{\partial \phi_A^{*2\prime}}$$

$$\frac{\partial F_1}{\partial \pi_A^{1\prime}} = \frac{\partial F_2}{\partial \pi_A^{2\prime}} \, . \tag{20}$$

Then the complete set of transformations (18) including both $a = 1$ and $a = 2$ will leave the two antibrackets invariant, preserving the complete triplectic anticanonical structure.

The constraints (20) restrict the possible dependence of the generators of these transformations on the variables $\phi_A^{*a\prime}$ and $\pi_A^{a\prime}$. Their general form can be written as

$$F_a = \mathbf{1}_a + f_a \tag{21}$$

with

$$\mathbf{1}_a = \phi^A \phi_{Aa}^{*\prime} + \bar{\phi}^A \pi_{Aa}^\prime$$

$$f_1 = g_1[\phi, \bar{\phi}] + g_3^A[\phi, \bar{\phi}]\pi_A^{1\prime} + g_4^A[\phi, \bar{\phi}]\phi_A^{*1\prime}$$

$$f_2 = g_2[\phi, \bar{\phi}] + g_3^A[\phi, \bar{\phi}]\pi_A^{2\prime} + g_4^A[\phi, \bar{\phi}]\phi_A^{*2\prime} \, , \tag{22}$$

where we have explicitly separated an identity operator $\mathbf{1}_a$ just for future convenience.

Now going back to the equation (18) we see that general triplectic canonical transformations can be put in the form

$$\phi^{\prime A} = \phi^A + g^[\phi, \bar{\phi}]$$

$$\phi_A^{*a} = \phi_A^{*a\prime} + \frac{\partial^r g_a}{\partial \phi^A}[\phi, \bar{\phi}] + \frac{\partial^r g_3^B}{\partial \phi^A}[\phi, \bar{\phi}]\pi_B^{a\prime} + \frac{\partial^r g_4^B}{\partial \phi^A}[\phi, \bar{\phi}]\phi_B^{*a\prime}$$

$$\bar{\phi}^{A\prime} = \bar{\phi}^A + g_3^A[\phi, \bar{\phi}]$$

$$\pi_A^a = \pi_A^{a\prime} + \frac{\partial^r g_a}{\partial \bar{\phi}^A}[\phi, \bar{\phi}] + \frac{\partial^r g_3^B}{\partial \bar{\phi}^A}[\phi, \bar{\phi}]\pi_B^{a\prime} + \frac{\partial^r g_4^B}{\partial \bar{\phi}^A}[\phi, \bar{\phi}]\phi_B^{*a\prime}$$

$$\tag{23}$$

The condition that a canonical transformation reproduces the gauge fixing corresponding to some boson B, after we express the result in terms of the transformed fields and impose the condition that $\bar{\phi}^{A\prime}$, $\phi_A^{*a\prime}$ and $\pi_A^{a\prime}$ are set to zero reads

6

$$\frac{\partial f'_a}{\partial \phi^A}\delta_a \phi^A \,+\, \frac{1}{2}g'_3 \delta_2\,\delta_1\phi^A \,-\, \frac{1}{2}\epsilon^{ab}\frac{\partial f'_a}{\partial \phi^A}\frac{\partial f'_b}{\partial \bar{\phi}^A} \;=\; \delta_2\delta_1 B[\phi^A]\,, \tag{24}$$

where we are defining the primed functions as the corresponding function, written in terms of ϕ^A and $\bar{\phi}^{A\prime}$, taken at $\bar{\phi}^{A\prime} = 0$

$$f'_a[\phi] \,=\, f_a[\,\phi\,,\,\bar{\phi}(\phi\,,\,\bar{\phi}'\,)\,]|_{\bar{\phi}'=0} \tag{25}$$

and a similar definition for g'_i.

Considering our W2 case, two illustrative possibilities are to choose

$$
\begin{aligned}
g_1 &= g_3 = g_4 = 0 \\
g_2 &= \delta_1\left(L(h - \tilde{h})\right) = L(\bar{\partial}c_1 - h\partial c_1 + \partial h c_1) + (\eta + \partial L c_1 - 2\partial l c_1)(h - \tilde{h}),
\end{aligned} \tag{26}
$$

or

$$
\begin{aligned}
g_1 &= -\delta_2\left(L(h - \tilde{h})\right) = L(\bar{\partial}c_2 - h\partial c_2 + \partial h c_2) + (-\bar{\eta} + \partial L c_2 - 2\partial l c_2)(h - \tilde{h}) \\
g_2 &= g_3 = g_4 = 0
\end{aligned} \tag{27}
$$

In both cases we get the gauge fixing action (17) if we perform the corresponding transformation in the fields of action S and then set all the primed antifields to zero.

5   One loop order

The first point that must be investigated is the possible effect of the introduction of the fields L , λ , η and $\bar{\eta}$ in the anomalies of the model. In other words, we must see if the cohomology of our extended formulation is the same as that from the original one. We can introduce a filtration[18] $\mathcal{N}$ that counts the number of fields and expand the BRST anti-BRST operators according to this filtration $\delta_1 = \delta_1^{(0)} + \delta_1^{(1)}$; $\delta_2 = \delta_2^{(0)} + \delta_2^{(1)}$. The first order piece of the algebra reads

$$
\begin{aligned}
\delta_1^{(0)}\eta &= 0 \\
\delta_1^{(0)}L &= \eta \\
\delta_1^{(0)}\lambda &= 0 \\
\delta_1^{(0)}\bar{\eta} &= \lambda \\
\delta_2^{(0)}\eta &= \lambda \\
\delta_2^{(0)}L &= -\bar{\eta} \\
\delta_2^{(0)}\lambda &= 0 \\
\delta_2^{(0)}\bar{\eta} &= 0\,.
\end{aligned} \tag{28}
$$

$$\tag{29}$$

Looking at this algebra we realize that this fields form doublets with respect to both the BRST and anti-BRST transformations. By a doublet one means a pair of fields say u, v whose transformations are of the form $\delta u = v$, $\delta v = 0$. Fields that show up just in doublets are absent from the cohomology of the BRST operator (or correspondingly homology of the anti BRST operator). Moreover, the cohomology ($a = 1$) or homology ($a = 2$) of the operator δ_a is contained in the cohomology (homology) of the corresponding $\delta_a^{(0)}$, $a = 1$. Thus we conclude that the inclusion of the fields L, λ, η and $\bar{\eta}$ does not change the cohomology (homology)of the W2 theory. We can then consider the same quantum correction ΔS to the first order master equations as for the standard formulation of W2. The calculation of ΔS depends on the introduction of a regularization procedure and the result depends on the result. But all the possible results differ just by trivial terms (in the cohomological sense). The simplest way to right the results of [19] adapting them to the extended symmetry is:

$$
\begin{aligned}
(\Delta^1 S)_{Reg} &= -\frac{1}{12\pi} \int d^2x \left(c_1 \, \partial^3 h \right) \\
(\Delta^2 S)_{Reg} &= -\frac{1}{12\pi} \int d^2x \left(c_2 \, \partial^3 h \right)
\end{aligned}
\tag{30}
$$

The presence of this term in the master equation at one loop order means a breaking in the BRST invariance. What one normally does in the BV quantization is then to introduce an extra Wess Zumino field θ representing the extra degree of freedom corresponding to the anomalous breaking of gauge invariance. The BRST extended version of this procedure would correspond to define this new field with the transformations:

$$
\begin{aligned}
\delta_1 \theta &= \partial c_1 + c_1 \partial \theta \\
\delta_2 \theta &= \partial c_2 + c_2 \partial \theta .
\end{aligned}
\tag{31}
$$

Then we verify that the counterterm

$$
M_1 = \frac{1}{24\pi} \int d^2x \left(\partial \theta \bar{\partial} \theta - h \partial \theta \partial \theta + h \partial^2 \theta \right)
\tag{32}
$$

solves the master equations:

$$
\{ S, M_1 \}^a + V^a M_1 = i (\Delta^a S)_{Reg}
\tag{33}
$$

for both $a = 1, 2$. As a remark we mention that a different approach could be taken to the addition of the field θ to the theory. We mean, one could include another ghost associated with the invariance of the classical action with respect to any transformations in θ. As a result the master equation would never be solved. One would only be able to shift the anomaly to this new symmetry[19], leaving the original gauge symmetry unbroken, but not the BRST (and anti-BRST symmetries). We will keep here the point of view of [20] that the field θ represents the new degree of freedom that shows up at quantum level as a consequence of the anomalous breaking of the original gauge symmetry. Thus we do not add any extra ghost.

Then we see that at one loop order the triplectic quantization reproduces the so called Wess Zumino mechanism of restoring the gauge invariance of an anomalous gauge theory by means of the introduction of an extra degree of freedom.

6   Conclusion

We have seen that, enlarging the space of fields, it is possible to formulate W2 gravity with extended BRST invariance. We have proven that these enlargement of the representation do not change the cohomology of the theory. We have calculated the one loop order corrections in the triplectic quantization for the model. We have seen that the anomalies and counterterms of the BRST and anti BRST sectors are essentially the same, up to changing ghosts by antighosts. The question that can then be raised then is: how general is this result? In the present case this happens because the BRST and anti BRST algebras are symmetric and thus one trivially concludes that the cohomology (homology) is the same for both symmetries (again, up to changing ghosts by antighosts). It seems an interesting future task to look, in some gauge theory, for a δ_2 symmetry satisfying the extended algebra $(\delta_2)^2 = \delta_1\delta_2 + \delta_2\delta_1 = 0$ but with a different homology.

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